A time-varying common risk factor affecting corporate yield spreads

Yusho KAGRAOKA

Musashi University, 1-26-1 Toyotama-kami, Nerima-ku, Tokyo 176-8534, Japan

Abstract

A time-varying common risk factor affecting corporate yield spreads is modelled by extending the panel data approach. This panel data model allows time-varying individual effects. The factor multiplied by a bond specific unobservable is identified as systematic risk premium. In disentangling the systematic risk premium, both credit and liquidity risks are evaluated; the credit risk is assessed by bond rating, and the liquidity risk is indirectly measured by discrepancy in quoted yields by brokerage firms. Parameters are estimated by the GMM procedure. The model is tested on the corporate bond market in Japan. Empirical results show that the time-varying common risk factor is successfully estimated together with credit and liquidity risk factors.

Key words: yield spread, systematic risk premium, panel data analysis

JEL Classification: C23, G12

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Email address: kagraoka@cc.musashi.ac.jp (Yusho KAGRAOKA).

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1 Introduction

Yields spreads of corporate bonds widen and tighten because of various reasons; changing risk tolerance of market participants, demand and supply in a corporate bond market, and so forth. When a majority of investors are reluctant to take any risk, the risk premium increases, and the yield spreads widen; on the other hand, when they are eager to take risk to enhance their investment performances, the risk premium decreases, and the yield spreads tighten. Yield spreads shrink in a period when relatively few bonds are issued, whereas yield spreads become greater when many bonds with large issue amounts are issued. Most corporate yield spreads change in the same direction, and their variations are correlated. Thus yield spreads are considered to be driven by bond specific factors and a marketwide one. We concentrate on the latter common factor, and we call a risk spread associated with the common factor as systematic risk premium.

Bond traders refer to a difference of yields between a corporate bond and the duration-equivalent Treasury bond as a credit spread. Investors assess the credit risk of corporate bonds using bond rating by rating agencies. However, there are other risk factors which are associated with a yield spread. One can decompose a yield spread into various components, and each of them is related to a risk factor. A yield spread of corporate bond reflects credit risk, liquidity risk, and transaction costs. Tax and commission fees constitute transaction costs. Studies of yield spreads at an instant of time cannot disentangle the systematic risk premium from credit and liquidity spreads. To isolate the systematic risk premium, evaluation of all kinds of risks is necessary. Measurement of the systematic risk premium is important to making decisions on
investment to corporate bonds since one cannot diversify the systematic risk in their bond portfolios. The systematic risk premium is regarded as an excess yield that can not be attributed to any bond-specific risks nor noise.

There are few papers on the systematic risk factors affecting corporate yield spreads. Fama and French (1993) find two common risk factors by regression analysis of corporate bond portfolios. The first factor, named TERM, is a difference between the one-month Treasury bill yield and an average yield of long-term government bonds. The second factor, named DEF, is a difference in the average yields between long-term corporate bonds and long-term government bonds. Their empirical results show that for most portfolios $R^2$ is over 0.90 except a low-rated portfolio of 0.49. Although they show that these two factors explain the corporate yield spreads, they do not discuss why the two factors model works well. Elton, Gruber, Agrawal and Mann (2001) examine yield spreads by considering three factors: loss from expected default, tax difference between corporate bonds and Treasury bonds, and systematic risk premium. They find that expected loss can account for no more than 25% of the corporate spreads, and a large portion of the yield spread is identified as risk premium which is explained by the Fama-French’s stock market factors. Gebhardt, Hvidkjaer, and Swaminathan (2005) elaborate the Fama-French model by incorporating rating and duration of bonds. Collin-Dufresne, Goldstein and Martin (2001) investigate determinants of yield spread changes. They consider numerous proxies which measure both changes in default probability and changes in recovery rate, as well as liquidity changes. They conclude that regression analysis can only explain 25% of the observed yield spread changes. Further they find that the residuals from these regressions are highly cross-correlated, and principal components analysis unveils that they are mostly
driven by a single common factor. However, they can not relate the common systematic factor to any macroeconomic variables.

The rest of studies on corporate yield spreads investigates the credit and the liquidity risks alone (Houweling, Mentink and Vorst (2005), Kagraoka (2005), Chen, Lesmond and Wei (2007), Longstaff, Mithal, and Neis (2005)). Elton, Gruber, Agrawal, and Mann (2004) report that tax liability, recovery rate, bond age, and default risk affect yield spreads. Huang and Huang (2003) study corporate bonds in a structural framework, and conclude that the credit risk accounts for a small fraction of the observed corporate yield spreads. Corporate bonds are traded mainly over-the-counter, and one cannot directly measure their liquidities. Data on bid-ask spreads are only available to brokerage firms. Thus we resort to an indirect approach to measure liquidity. Various proxies to the liquidity have been proposed in the literatures. One of the latest studies on liquidity proxies is conducted by Houweling, Mentink and Vorst (2005). They examine nine proxies (issued amount, listed, euro, on-the-run, age, missing prices, yield volatility, number of contributors and yield dispersion). They conclude that eight proxies (except listed) can be taken as liquidity measures. The growing market of credit derivatives allows us to directly measure the liquidity spreads of corporate bonds. Credit default swaps are one of the typical products of credit derivatives. Credit derivatives are regarded as having no liquidity risk since the notional amount of credit default swaps can be arbitrarily large. If a party wants to close a credit default swap position, he simply enters a new swap in the opposite direction. Longstaff, Mithal, and Neis (2005) compare premium of credit default swaps and yield spreads of corporate bonds, and extract the non-credit component. The credit default swap is evaluated using a reduced form model. They find that the de-
fault component represents 51% of the spread for AAA/AA-rated bonds, 56% for A-rated bonds, 71% for BBB-rated bonds, and 83% for BB-rated bonds. The non-default component ranges from 20 to 100 basis points.

In this paper, we study the time-varying common risk factor as well as the systematic risk premium, the credit and the liquidity spreads. No studies investigates the time-varying common risk factor which simultaneously affects all corporate yield spreads. We conjecture that this risk factor is the unidentified common factor in Collin-Dufresne, Goldstein and Martin (2001). Our common risk factor is related to the DEF factor in Fama and French (1993). To disentangle the common risk factor from the credit and the liquidity risks, we quantitatively evaluate both risks. The credit risk is assessed by bond rating. The liquidity risk is measured by issued amount of a corporate bond, and yield discrepancy in quoted yields by brokerage firms. Previous studies do not employ time series analysis of yield spreads, nor panel data analysis. Ahn, Lee and Schmidt (2001) develop a panel data model with time-varying individual effects. They present parameter estimation procedures based on the Generalized Methods of Moments (GMM). This model enables us to model the systematic risk premium of corporate bonds in a direct way. We conduct an empirical study on corporate yield spreads, and estimate the systematic risk premium following the method of Ahn, Lee and Schmidt (2001). The goal of the statistical analysis is to uncover the basic structure in the yield spreads, and to isolate the separate effects of time-varying common factor. Contribution of the paper is multifold. First, we identify the systematic risk premium of corporate bonds. Secondly, we decompose yield spread into the systematic risk premium, the credit component, and the liquidity component. Thirdly, we propose the yield discrepancy as a liquidity proxy. Fourthly, we implement
the panel data model with time-varying individual effects. This model makes us possible to capture dynamics of yield spread. Fifthly, we conduct empirical analysis of Japanese corporate bonds.

The remainder of the paper is organized as follows. Section 2 reviews the panel data model with time-varying individual effects. Our model and explanatory variables are also illustrated. Section 3 describes our data and presents empirical results. Finally, Section 4 summarizes the paper and gives discussions on our model.

2 Panel data analysis of yield spreads

2.1 Panel data model

We apply an extended version of the fixed-effects model to treat time varying yield spreads. Ahn, Lee and Schmidt (2001) develop the Generalized Methods of Moments (GMM) estimation of the fixed-effect model in which the individual effects are time varying. To make the paper self-contained, we briefly review their model and their estimation procedure. We follow their notation as possible.

We define a yield spread of the \(i\)th corporate bond at time \(t\) by

\[
s_{it} = R_{it} - R_{f,it},
\]

where \(R_{it}\) and \(R_{f,it}\) are yields of the \(i\)th corporate bond and that of the duration-equivalent Treasury bond, respectively. One can decompose a yield spread into systematic risk premium, credit spread, liquidity spread, and trans-
action costs. We assume that the credit and the liquidity risk factors are independent, and that they contribute to a yield spread multiplicatively. The logarithm of the $i$th bond’s yield spread

$$y_{it} = \ln s_{it},$$

is represented as

$$y_{it} = X_{it}\beta + Z_{i}\gamma + u_{it},$$

$$u_{it} = \theta_t\alpha_i + \epsilon_{it},$$

$$(i = 1, \ldots, N, \ t = 1, \ldots, T).$$

Here $X_{it}$ is a $1 \times k$ vector of time-varying explanatory variables, and $Z_{i}$ is a $1 \times g$ vector of time-invariant regressors. The last entry of $Z_{i}$ is one, so that the last parameter in $\gamma$ denotes the overall intercept term. The $\epsilon_{it}$ are random noise with $E[\epsilon_{it}] = 0$. The parameter $\alpha_{i}$ are unobservables, and $\theta_{t}$ is the parameter measuring the effect of $\alpha_{i}$ on $y_{it}$ at time $t$. Changes of yield spreads are different depending on $\alpha_{i}$. We can interpret $\theta_{t}$ as a common risk factor affecting all yield spreads, and $\theta_{t}$ multiplied by $\alpha_{i}$ as the systematic risk premium to the $i$th corporate bond, respectively. The factor $\theta_{t}$ drives co-movement of corporate yield spreads. The unobserved parameter $\alpha_{i}$ represents a bond-specific factor loading.

We discuss parameter estimation of the model. To identify the model, we set $\theta_{1} = 1$. We introduce notations, $\theta = (\theta_{2}, \theta_{3}, \ldots, \theta_{T})'$, and $\xi = (1, \theta')'$, and we rewrite the $T$ observations for the $i$th bond in a matrix form as

$$y_{i} = X_{i}\beta + e_{T}Z_{i}\gamma + u_{i},$$

7
\begin{equation}
u_i = \xi \alpha_i + \epsilon_i, \tag{6}\end{equation}

where $e_T$ is the $T \times 1$ vector of ones, $y_i = (y_{i1}, y_{i2}, \ldots, y_{iT})'$, and $X_i$ and $u_i$ are similarly defined. We bind up all the explanatory variables into

\begin{equation}W_i = (X_{i1}, X_{i2}, \ldots, X_{iT}, Z_i). \tag{7}\end{equation}

We define a covariance matrix,

\begin{equation}\Sigma = E[(W_i, \alpha_i)'(W_i, \alpha_i)] = \begin{bmatrix} \Sigma_{ww} & \Sigma_{w\alpha} \\ \Sigma_{\alpha w} & \Sigma_{\alpha\alpha} \end{bmatrix}. \tag{8}\end{equation}

To identify the model, we impose the following assumptions (called the basic assumptions in Ahn, Lee and Schmidt (2001));

1. $[W_i, \alpha_i, \epsilon_i']$ is independently and identically distributed over $i$.
2. $\epsilon_{it}$ has finite moments up to forth order, and $E[\epsilon_{it}] = 0$.
3. The second moment matrix $\Sigma$ is finite and nonsingular.
4. $E[W'_i(Z_i, \alpha_i)]$ is of full column rank.
5. $[W'_i, \alpha_i]$ is uncorrelated with $\epsilon_i$.

We define a triplet of the parameters of interest as $\delta = (\beta', \gamma', \theta')'$, and denote the resulting estimate by $\hat{\delta}$. Ahn, Lee and Schmidt (2001) establish estimation procedures by the GMM. A standard GMM estimate is obtained from the moment conditions,

\begin{equation}E[b_i(\delta)] = E[W'_i(u_{it} - \theta_i u_{i1})] = 0, \quad t = 2, 3, \ldots, T. \tag{9}\end{equation}

We define the sample average of $b_i$ by

\begin{equation}b_N = \frac{1}{N} \sum_{i=1}^{N} b_i(\delta). \tag{10}\end{equation}
Then, the optimal GMM estimator of $\hat{\delta}$ solves the problem

$$\min_{\delta} N b_N(\delta)' V^{-1} b_N(\delta), \quad (11)$$

where

$$V = E[b_i(\delta) b_i(\delta)']. \quad (12)$$

The asymptotic covariance matrix of $\sqrt{N}(\hat{\delta} - \delta)$ equals

$$[B'(V)^{-1} B]^{-1}, \quad (13)$$

where

$$B = E \left( \frac{\partial b_i}{\partial \delta} \right). \quad (14)$$

2.2 Explanatory variables

In this subsection, we explain our explanatory variables adopted in the panel data model. The explanatory variables are summarized in Table 1. We have three types of parameters in eqs. (3) and (4); $\beta$ is the coefficient to the dynamic variable $X_{it}$, $\gamma$ to the static variable $Z_i$, and $\theta_t$ to the unobservable $\alpha_i$, respectively. We take the yield of 10-year Japanese Government Bonds (JGBs) as the risk-free rate. The 10-year JGBs are most liquid and work as a benchmark in the Japanese bond market. To investigate the effect of $\theta_t$, all the risk entering yield spreads should be taken into account. Commission fees and taxes are the same for Treasury bonds and corporate bonds in Japan, and their effects to yield spreads cancel out in the calculation of yield spreads. We first discuss the credit risk, and subsequently the liquidity risk.
Following many previous studies (Gebhardt, Hvidkjaer, and Swaminathan (2005), Houweling, Mentink and Vorst (2005), Collin-Dufresne, Goldstein and Martin (2001), Kagraoka (2005)), we adopt rating of corporate bonds to assess the credit risk. We employ the bond rating reported by R&I, a major rating company in Japan. Rating of a corporate bond sometimes changes, however it does not change frequently. It is difficult to estimate $\beta$ to the credit rating unless we have long history of corporate yield spreads with changing rating classes. Because of the limitation of the history of corporate yields in our dataset, we restrict ourselves to corporate bonds whose ratings are the same through the entire time period (form $t = 1$ to $t = T$), and we regard the rating as a static variable. Then the rating of bonds is an entry to the time-invariant regressor $Z_i$. We analyze the investment grade bonds (rating at least BBB−) since speculative-grade bonds are sometimes improperly priced. There are few bonds in the highest rating classes, as well as in the lowest rating classes. We aggregate some rating classes, and we have six categories: AAA/AA+/AA, AA−, A+, A, A−, and BBB+/BBB/BBB−. We introduce dummy variable to each rating category except the highest rating category. Coefficient to a dummy variable is a spread to the highest rating category. We take AAA/AA+/AA as the baseline, and the coefficients to the rating dummies are spreads to AAA/AA+/AA.

As for liquidity proxies, we incorporate issued amount into our explanatory variables as Houweling, Mentink and Vorst (2005) and Kagraoka (2005). We take the logarithm of issued amount (in billion yen) because the liquidity dose not have linear dependence with the issued amount. The issued amount of a bond is constant, and the logarithm of the issued amount is an element of $Z_i$. Houweling, Mentink and Vorst (2005) propose many proxies besides the issue
amount, however their proxies are not appropriate in our case; some proxies are useless in the Japanese bond market, and rest of them are not recorded in our dataset. We introduce a new proxy, yield discrepancy, which is a difference between the highest and the lowest of quoted yields by brokerage firms. If a corporate bond is liquid, quoted prices by brokerage firms are very close to each other. If a bond is illiquid, quoted prices by brokerage firms straggle out. Therefore we expect that the yield discrepancy is greater for less liquid bonds. The yield discrepancy is a dynamic variable, and it is an element of time dependent explanatory variables $X_{it}$.

3 Empirical analysis

3.1 Data

Data are provided by the Japan Securities Dealers Association (JSDA). The JSDA have published the reference yields for over-the-counter bond transactions from August 2002. The yields designate in order to guide the JSDA members and customers in the over-the-counter based transactions. The reference yields is calculated by the JSDA based on quotations reported by the designated-reporting members of the JSDA. As of April 2006, the JSDA nominates 21 major securities companies as the designated-reporting members. The reference yields are statistical summary of quotations: the arithmetical average yield, the median, the highest yield, and the lowest yield. Quotation is a mid yield for buys and sells. The JSDA publishes the number of securities companies which report mid yield quotations. The JSDA calculates the statistics after they eliminate some of the highest and the lowest yields in
quotations by the designated-reporting members. The number of coverage of corporate bonds is about two thousands.

Our dataset records daily reference yields from October 1, 2002 to December 30, 2004. We choose the corporate bonds which are rated by R&I, a major Japanese rating company. We select investment grade bonds (whose ratings are at least BBB−) since sometimes junk bonds are priced in a subtly manner. To regard the rating class as a static variable, we reject corporate bonds whose ratings change in the data period. Further we filter corporate bonds based on the following criteria:

- Coupon rate is fixed, and coupons are paid semi-annually.
- Principal amount is fully repaid at the maturity.
- The bond has no call provision.
- The bond is unsecured, and it is not subordinated.
- There is no missing yields in the data period.
- Remaining term to maturity of the bond is greater than one year at December 30, 2004.

Finally, we have 340 corporate bonds. The number of bonds by rating class is summarized in Table 2. Time series of average of the logarithm of the yield spread by the rating category are depicted in Fig. 1.

In our panel data analysis, we pick up quarterly dates because we observe that changes of yield spreads are rather slow. The first date in the panel data analysis is October 31, 2002, and we have nine dates up to December 30, 2004. In the period, yield spreads gradually shrink; the yields range from 3.621% to 0.081%, and their yield spreads range from 3.397% to 0.013%. At the last date, the yield spread is very tight. Descriptive statistics on the yields, the
logarithm of the yield spreads, and the liquidity proxies are given in Table 3.

3.2 Empirical results

We examine our empirical result by the GMM estimation. The results are presented in Table 4. All the coefficients in the panel data model are economically and statistically significant.

First we examine the time-varying factors, from $\theta_2$ to $\theta_9$. As for identification, we set $\theta_1 = 1$. The coefficient $\theta_t$ becomes smaller as time passes in accordance with tightening of the yield spreads; at the last date $\theta_9$ takes 0.329604. We depict a time series of $\theta_t$ in Fig. 2. The trajectory of $\theta_t$ is in accordance with the declining of the average of the yield spreads (see Fig. 1). The $\theta_t$ contributes to the yield spread with a factor loading $\alpha_i$, and the quantity $\theta_t\alpha_i$ is identified as the systematic risk premium. The yield spreads become tight in the period, and $\theta_t$ is positive and decreasing. Then we conclude that the unobservables $\alpha_i$ are positive.

Next we look at the parameter concerning the credit risk. As for the rating category, the baseline corresponds to AAA/AA+/AA. The estimated coefficients are 0.072957 for AA−, 0.208515 for A+, 0.259647 for A, 0.333516 for A−, and 0.395840 for BBB+/BBB/BBB−, respectively. The coefficients to the bond rating are greater than zero, and they gradually increase as their ratings deteriorate. The magnitudes of the estimated parameter and their ordering by the rating category are consistent. Subsequently we investigate the coefficients related to the liquidity. The coefficient to the yield discrepancy is 0.749261. This fact means that the bigger the yield discrepancy is, the greater the yield
spread becomes. The positive parameter is consistent with our expectation that the yield discrepancy is proportional to the liquidity. The coefficient to the logarithm of issued amount is negative of $-0.066522$. This means that the bigger issued amount improves the liquidity of a corporate bond. Judging from the magnitudes of the coefficients, we consider that the yield discrepancy is a better proxy to the liquidity than the logarithm of the issued amount.

To summarize our results, we are successful to estimate the systematic risk premium. The credit risk is evaluated by the bond rating. The liquidity risk is measured by the yield discrepancy and the logarithm of issued amount.

4 Conclusion

In this paper, we identify the time-varying common factor affecting corporate yield spreads. We decompose a yield spread into the systematic risk premium, the credit component, and the liquidity component. We implement the panel data model with time-varying individual effects, and we conduct empirical analysis of Japanese corporate bonds. In the estimation, both the credit risk and the liquidity risk are evaluated. We find that the common risk factor changes in accordance with a change of the yield spreads. The credit risk is properly assessed by the bond rating. The liquidity risk is measured by the yield discrepancy in quoted yields by brokerage firms. The logarithm of the issued amount also works as a liquidity proxy.

Collin-Dufresne, Goldstein and Martin (2001) investigate the determinants of corporate yield spread changes by employing simple regression of yield spread. They find the residuals from the regression are highly cross-correlated, and
mostly driven by a single common factor. They fail to explain the factor, and conjecture that yield spread changes are principally driven by local supply/demand shocks that are independent of both credit-risk factors and standard proxies for liquidity. In our model, the risk premium is decomposed to the global time-varying factor $\theta_t$ and the bond specific sensitivity $\alpha_i$. The global factor is common to the bonds, and we conjecture that $\theta_t$ is the common risk factor found by Collin-Dufresne, Goldstein and Martin (2001). Our result support the presence of the DEF factor in Fama and French (1993).

We adopt only two liquidity proxies in our empirical analysis because our data sample period is not long, and it might be difficult to get robust results if we would take more liquidity proxies in our model. This does not mean a limitation of our model. If we have a large dataset on corporate yield spreads, we can incorporate other liquidity proxies such as age of bond, duration, bid-ask spreads, and so on.

We assume that the risk premium is a linear scalar function as $\theta_t \alpha_i$ in equations (3) and (4). We extend the model to incorporate nonlinear systematic risk premium. Han, Orea and Schmidt (2005) extend Ahn, Lee and Schmidt (2001) where time-varying individual effects are parametric function of time-varying coefficients of individual effects.

$$y_{i,t} = X_{it} \beta + Z \gamma_i + \lambda(\theta) \alpha_i + \epsilon_{i,t}. \quad (15)$$

We can extend risk premium to multi-factor by applying the panel data model proposed by Bai (2005).
References


Bai, J., 2005, Panel data models with interactive fixed effects, working paper, New York University.


Huang, J., and M. Huang, 2003, How much of the corporate-Treasury yield
spread is due to credit risk?, working paper, Stanford University.


Fig. 1. Logarithm of the yield spread.

Time series of yield spreads is depicted. Yield spreads are measured in percent. We take average of the logarithm of the yield spreads by bond rating.
The logarithm of the $i$th bond’s yield spread, $y_{it}$, is modelled by

$$ y_{it} = X_{it} \beta + Z_i \gamma + u_{it}, $$

$$ u_{it} = \theta_t \alpha_i + \epsilon_{it}. $$

The time-varying common factor $\theta_t$ is depicted. $\theta_1$ is set to 1 for identification.
Table 1
List of the explanatory variables.

<table>
<thead>
<tr>
<th>( \beta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>yield discrepancy</td>
</tr>
<tr>
<td>discrepancy in quoted yields by dealers; difference between the highest and the lowest of quoted yields by dealers</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( \gamma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>issued amount</td>
</tr>
<tr>
<td>logarithm of the issued amount of a corporate bond (in billion yen)</td>
</tr>
<tr>
<td>AAA/AA+/AA</td>
</tr>
<tr>
<td>rating of corporate bond, which corresponds baseline of the model</td>
</tr>
<tr>
<td>AA(^-)</td>
</tr>
<tr>
<td>taking 1 if rating of corporate bond is AA(^-), otherwise 0</td>
</tr>
<tr>
<td>A+</td>
</tr>
<tr>
<td>taking 1 if rating of corporate bond is A+, otherwise 0</td>
</tr>
<tr>
<td>A</td>
</tr>
<tr>
<td>taking 1 if rating of corporate bond is A, otherwise 0</td>
</tr>
<tr>
<td>A(^-)</td>
</tr>
<tr>
<td>taking 1 if rating of corporate bond is A(^-), otherwise 0</td>
</tr>
<tr>
<td>BBB+/BBB/BBB(^-)</td>
</tr>
<tr>
<td>taking 1 if rating of corporate bond is either BBB(+), BBB, or BBB(^-), otherwise 0</td>
</tr>
<tr>
<td>intercept</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( \theta_t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \theta_2, \theta_3, \ldots, \theta_T )</td>
</tr>
<tr>
<td>factor loading to time-varying individual effects</td>
</tr>
</tbody>
</table>

The logarithm of the \( i \)th bond’s yield spread, \( y_{it} \), is modelled by

\[
y_{it} = X_{it} \beta + Z_i \gamma + u_{it},
\]

\[
u_{it} = \theta_t \alpha_i + \epsilon_{it}.
\]

\( \beta \) is coefficient to dynamic explanatory variables, and \( \gamma \) is that to static explanatory variables. \( \theta_t \) is a time-varying common factor.
Table 2
The number of corporate bonds by rating class.

<table>
<thead>
<tr>
<th>AAA</th>
<th>AA+</th>
<th>AA</th>
<th>AA−</th>
<th>A+</th>
<th>A</th>
<th>A−</th>
<th>BBB+</th>
<th>BBB</th>
<th>BBB−</th>
<th>total</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>22</td>
<td>28</td>
<td>52</td>
<td>49</td>
<td>69</td>
<td>67</td>
<td>31</td>
<td>13</td>
<td>5</td>
<td>340</td>
</tr>
</tbody>
</table>
Table 3
Descriptive statistics of the yield spread and the liquidity proxies.

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Median</th>
<th>Maximum</th>
<th>Minimum</th>
<th>Std. Dev.</th>
<th>Skewness</th>
<th>Kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>yield spread (%)</td>
<td>0.3674</td>
<td>0.2575</td>
<td>3.3971</td>
<td>0.0130</td>
<td>0.3461</td>
<td>3.4422</td>
<td>20.2221</td>
</tr>
<tr>
<td>log of yield spread</td>
<td>-1.2676</td>
<td>-1.3566</td>
<td>1.2229</td>
<td>-4.3419</td>
<td>0.6824</td>
<td>0.6035</td>
<td>3.4154</td>
</tr>
<tr>
<td>yield discrepancy (%)</td>
<td>0.1302</td>
<td>0.1100</td>
<td>1.0110</td>
<td>0.0060</td>
<td>0.0942</td>
<td>2.1481</td>
<td>11.5865</td>
</tr>
<tr>
<td>log of issued amount</td>
<td>2.5840</td>
<td>2.3026</td>
<td>5.2983</td>
<td>1.0986</td>
<td>0.6639</td>
<td>0.8695</td>
<td>4.9323</td>
</tr>
</tbody>
</table>

Issue amount is expressed in billion yen.
Table 4  
Empirical results of panel data analysis.

<table>
<thead>
<tr>
<th></th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>yield discrepancy</td>
<td>0.749261</td>
<td>0.018423</td>
<td>40.670490</td>
<td>0.000000</td>
</tr>
<tr>
<td>log(amount)</td>
<td>-0.066522</td>
<td>0.014977</td>
<td>-4.441703</td>
<td>0.000000</td>
</tr>
<tr>
<td>AA−</td>
<td>0.072957</td>
<td>0.027342</td>
<td>2.668275</td>
<td>0.007700</td>
</tr>
<tr>
<td>A+</td>
<td>0.208515</td>
<td>0.025839</td>
<td>8.069737</td>
<td>0.000000</td>
</tr>
<tr>
<td>A</td>
<td>0.259647</td>
<td>0.038770</td>
<td>6.697129</td>
<td>0.000000</td>
</tr>
<tr>
<td>A−</td>
<td>0.333516</td>
<td>0.038485</td>
<td>8.666066</td>
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<td>BBB+/BBB/BBB−</td>
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<td>0.044878</td>
<td>8.820268</td>
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<td>-51.757160</td>
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<tr>
<td>θ2</td>
<td>0.845203</td>
<td>0.004694</td>
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<td>θ3</td>
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<td>θ4</td>
<td>0.768707</td>
<td>0.006642</td>
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<td>θ5</td>
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<td>θ9</td>
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</table>

The logarithm of the $i$th bond’s yield spread, $y_{it}$, is modelled by

$$y_{it} = X_{it} \beta + Z_{i} \gamma + u_{it},$$

$$u_{it} = \theta_{t} \alpha_{i} + \epsilon_{it}.$$  

The regression coefficients $\beta$ and $\gamma$, and time varying coefficient $\theta_{t}$ are estimated. The parameter $\theta_{1}$ is normalized to 1, and $t$-static for $\theta_{t}$’s are irrelevant.